

# The decays $\bar{B} \rightarrow \bar{K}D$ and $\bar{B} \rightarrow \bar{K}\bar{D}$ and final state interactions

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The decays  $\bar{B} \rightarrow \bar{K}D$  and  $\bar{B} \rightarrow \bar{K}\bar{D}$  taking into account final state interactions are discussed. These decays are described by four strong phases  $\delta_0, \delta_1, \bar{\delta}_0, \bar{\delta}_1$  (subscripts 0 and 1 refers to  $I = 0$  and  $I = 1$  final states), one weak phase  $\gamma$  and four real amplitudes. It is argued that strong interaction dynamics implies  $\bar{\delta}_1 = 0, \delta_0 = -\delta_1$ . Rescattering has significant effects on weak amplitudes. Taking into account, rescattering, we find that direct CP-violating asymmetry in these decays may lie in the range  $\mp 0.023 \sin \gamma \leq \mathcal{A}_{1,2} \leq \mp 0.086 \sin \gamma$ .

The weak decays  $\bar{B} \rightarrow \bar{K}D$  and  $\bar{B} \rightarrow \bar{K}\bar{D}$  taking into account final state interactions have been studied by several authors [1–4]. These decays are described by four real amplitudes, four strong phases  $\delta_0, \delta_1, \bar{\delta}_0, \bar{\delta}_1$  and one weak phase  $\gamma$ . The effective Lagrangian which describes these decays are given by

$$L_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* [\bar{s} \gamma^\mu (1 - \gamma_5) u] [\bar{c} \gamma_\mu (1 - \gamma_5) b] \quad (1a)$$

$$L_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub} V_{cs}^* [\bar{s} \gamma^\mu (1 - \gamma_5) c] [\bar{u} \gamma_\mu (1 - \gamma_5) b] \quad (1b)$$

Since the effective weak Lagrangians for these decays have  $\Delta I = 1/2$ , the isospin analysis [4] for  $\bar{K}D$  :

$$A(B^- \rightarrow K^- D^0) = 2f_1 e^{i\delta_1} \quad (2a)$$

$$A(\bar{B}^0 \rightarrow K^- D^+) = [f_1 e^{i\delta_1} + f_0 e^{i\delta_0}] \quad (2b)$$

$$A(\bar{B}^0 \rightarrow \bar{K}^0 D^0) = [f_1 e^{i\delta_1} - f_0 e^{i\delta_0}] \quad (2c)$$

where  $\delta_0$  and  $\delta_1$  are the phase shifts for  $I = 0$  and  $I = 1$  isospin states. On the other hand for  $\bar{K}\bar{D}$  we have

$$A(\bar{B}^0 \rightarrow \bar{K}^0 \bar{D}^0) = 2\bar{f}_1 e^{i\gamma} e^{i\bar{\delta}_1} \quad (3a)$$

$$A(B^- \rightarrow K^- \bar{D}^0) = e^{i\gamma} [\bar{f}_1 e^{i\bar{\delta}_1} + \bar{f}_0 e^{i\bar{\delta}_0}] \quad (3b)$$

$$A(B^- \rightarrow \bar{K}^0 D^-) = e^{i\gamma} [-\bar{f}_1 e^{i\bar{\delta}_1} + \bar{f}_0 e^{i\bar{\delta}_0}] \quad (3c)$$

In addition to above decays, other decays relevant to our analysis are

$$A(\bar{B}^0 \rightarrow \pi^+ D_s^-) = \bar{f} e^{i\gamma} e^{i\bar{\delta}} = \sqrt{2} A(B^- \rightarrow \pi^0 D_s^-) \quad (4)$$

$$A(B^- \rightarrow \eta D_s^-) = \frac{1}{\sqrt{6}} e^{i\gamma} [\bar{f} e^{i\bar{\delta}} - 2(-\bar{f}_1 e^{i\bar{\delta}_1} + \bar{f}_0 e^{i\bar{\delta}_0})] \quad (5)$$

where in writing Eq. (5), we have used SU(3) which gives

$$\sqrt{6} A(B^- \rightarrow \eta D_s^-) = A(\bar{B}^0 \rightarrow \pi^+ D_s^-) - 2A(B^- \rightarrow \bar{K}^0 \bar{D}) \quad (6)$$

In this paper, we address two questions: Is it possible to get some constraints on strong phases ? How the weak amplitudes are affected by rescattering ? In answer to first question, our analysis implies

$$\delta_0 = -\delta_1, \quad \bar{\delta}_1 = 0. \quad (7)$$

We find that rescattering has significant effect on some observables for these decays.

Let us consider the scattering processes:

$$\begin{aligned} \bar{K} + D &\rightarrow \bar{K} + D \\ \bar{K} + \bar{D} &\rightarrow \bar{K} + \bar{D} \end{aligned}$$

where  $\bar{K} \equiv (\bar{K}^0, K^-)$ ,  $D \equiv (D^+, D^0)$ ,  $\bar{D} \equiv (\bar{D}^0, D^-)$ . We note that in  $s$  or  $u$ -channel only the states with quark structure  $s\bar{c}$  can be exchanged ( $s, u$  and  $t$  are Mandelstam variables). An important consequence of this is that  $s$ -channel is exotic for  $\bar{K}D$  scattering whereas  $u$ -channel is exotic for  $\bar{K}\bar{D}$  scattering. Since  $s\bar{c}$  state has isospin

$I = 0$ , this exchange in  $s$ -channel gives isospin projection operator  $-2P_0$  where as  $I = 0$  exchange in  $u$ -channel gives isospin projection operator  $P_1 - P_0$ . In  $t$ -channel,  $I = 1$  ( $\rho$ ) exchange gives an isospin factor  $P_1 - 3P_0$  where as  $I = 0$  ( $\omega$ ) exchange gives an isospin factor  $P_1 + P_0$ . Assuming  $\rho - \omega$  degeneracy,  $I = 1$  and  $I = 0$  exchanges give  $(P_1 - 3P_0) + (P_1 + P_0) = 2(P_1 - P_0)$  isospin projection operator for  $\bar{K}D$  scattering where as for  $\bar{K}\bar{D}$  scattering. we get  $(P_1 - 3P_0) - (P_1 + P_0) = -4P_0$ . Hence for  $\bar{K}D$  and  $\bar{K}\bar{D}$ , the scattering amplitudes can be written as

$$M = -gA(P_1 - P_0) \quad (8)$$

$$\bar{M} = -g\bar{A}(-2P_0) \quad (9)$$

Eqs. (8) and (9) imply  $\delta_0 = -\delta_1, \bar{\delta}_1 = 0$  viz Eq. (7).

It is clear from Eq. (8) that scattering matrices for  $\bar{K}D$  for  $Q = -1$  and  $Q = 0$  states are given by respectively

$$M_{-0} = -gA, \quad (K^- D^0 \rightarrow K^- D^0) \quad (10)$$

$$M = \begin{array}{c|cc} & 00 & -+ \\ \hline 00 & 0 & -g\bar{A} \\ -+ & -gA & 0 \end{array} \quad (11)$$

From Eq. (8), we get for  $Q=0$  and  $Q=-1$  states of  $\bar{K}\bar{D}$ , the scattering matrices

$$\bar{M}_{00} = 0, \quad (\bar{K}^0 \bar{D}^0 \rightarrow \bar{K}^0 \bar{D}^0) \quad (12)$$

$$\bar{M} = \begin{array}{c|cc} & 0- & -0 \\ \hline 0- & g\bar{A} & -g\bar{A} \\ -0 & -g\bar{A} & g\bar{A} \end{array} \quad (13)$$

However, we note that for  $\bar{K}\bar{D}$  scattering other channels  $\pi^+ D_s^-, \pi^0 D_s^-$  and  $\eta D_s^-$  are also open. Hence we must extend the scattering matrices given in Eqs. (12) and (13) to take into account the inelastic channels. Using SU(3), we get the scattering matrices

$$\bar{M}(Q=0) = \begin{array}{c|cc} & 00 & \pi^+ s^- \\ \hline 00 & 0 & -g\bar{A} \\ \pi^+ s^- & -gA & 0 \end{array} \quad (14)$$

and

$$\bar{M}(Q=-1) = \begin{array}{ccccc} & 0- & -0 & \pi^0 s^- & \eta s^- \\ & g\bar{A} & -g\bar{A} & \frac{1}{\sqrt{2}}gA & \frac{1}{\sqrt{6}}g(2\bar{A}-A) \\ 0- & -g\bar{A} & g\bar{A} & -\frac{1}{\sqrt{2}}gA & \frac{1}{\sqrt{6}}g(2\bar{A}-A) \\ \pi^0 s^- & \frac{1}{\sqrt{2}}gA & -\frac{1}{\sqrt{2}}gA & 0 & 0 \\ \eta s^- & \frac{1}{\sqrt{6}}g(2\bar{A}-A) & \frac{1}{\sqrt{6}}g(2\bar{A}-A) & 0 & -\frac{2}{3}g(A+\bar{A}) \end{array} \quad (15)$$

To proceed further we note that in terms of Regge phenomenology, exotic  $u$ -channel implies exchange degeneracy i.e. in  $t$ -channel  $\rho - A_2$  as well as  $\omega - f$  trajectories are exchange degenerate. Taking this into account, it is convenient to express amplitudes  $A$  and  $\bar{A}$  in Veneziano representation [5]

$$A = \left[ \frac{\Gamma(1 - \alpha_{D_s^*}(u)) \Gamma(1 - \alpha(t))}{\Gamma(1 - \alpha_{D_s^*}(u) - \alpha(t))} \right] \quad (16)$$

$$\bar{A} = \left[ \frac{\Gamma(1 - \alpha_{D_s^*}(s)) \Gamma(1 - \alpha(t))}{\Gamma(1 - \alpha_{D_s^*}(s) - \alpha(t))} \right] \quad (17)$$

We will take linear Regge trajectories viz

$$\begin{aligned} \alpha &= \alpha_0 + \alpha' t \\ \alpha_{D_s^*}(u) &= \alpha_{D_s^*}(0) + \alpha' u \\ \alpha_{D_s^*}(s) &= \alpha_{D_s^*}(0) + \alpha' s \end{aligned} \quad (18)$$

We assume universal slope viz

$$\alpha' \simeq 0.94 \text{ GeV}^{-2} \approx \frac{1}{s_0} \quad (19)$$

and take  $\alpha_0 = 0.46$ . In actual numerical evaluation we will put  $\alpha_0 = 1/2$  and  $s_0 = 1 \text{ GeV}^2$ . Note that  $\alpha_{D_s^*}(0) = 1 - \alpha' m_{D_s^*}^2$  for large  $s$ , we get from Eqs. (16) and (17):

$$A \rightarrow \frac{\pi}{\sin \pi \alpha(t) \Gamma(\alpha(t))} \left( \frac{s}{s_0} \right)^{\alpha(t)} \quad (20)$$

$$\bar{A} \rightarrow \frac{\pi e^{-i\pi\alpha(t)}}{\sin \pi \alpha(t) \Gamma(\alpha(t))} \left( \frac{s}{s_0} \right)^{\alpha(t)} \quad (21)$$

From Eqs. (17), we note that  $1 - \alpha_{D_s^*}(s) = 0$  gives a pole at  $s = m_{D_s^*}^2$  and  $1 - \alpha(t) = 0$  gives a pole at  $t = m_\rho^2$ . Using this property of Veneziano representation, we get

$$-g = 2g_{D_s^* DK}^2 = 4g_{\rho K \bar{K}} g_{\rho D \bar{D}} \quad (22)$$

Using the usual parameterization for  $g_{D_s^* DK}^2$ , we get

$$g_{D_s^* DK}^2 = \gamma_D^2 \frac{m_{D_s^*}^2}{f_K^2} \simeq 50 \quad (23)$$

for  $\gamma_D = 1/2$ .

We now come to the second question viz the effect of rescattering on the weak amplitudes. First we discuss the rescattering for  $\bar{K}D$  system. Two particle unitarity gives [6–8]

$$\begin{aligned} \text{Disc } A(\bar{B}^0 \rightarrow \bar{K}^0 D^0) &= \frac{1}{32\pi} \frac{|\mathbf{p}|}{s} \int d\Omega A(\bar{B}^0 \rightarrow K^- D^+) M^*(K^- D^+ \rightarrow \bar{K}^0 D^0) \\ &= \frac{1}{32\pi} \frac{1}{\sqrt{s} |\mathbf{p}|} \int_{-2|\mathbf{p}|^2}^0 dt A(\bar{B}^0 \rightarrow K^- D^+) M^*(K^- D^+ \rightarrow \bar{K}^0 D^0) \end{aligned} \quad (24)$$

Using Eqs. (11) and (20), we get

$$\text{Disc } A(\bar{B}^0 \rightarrow \bar{K}^0 D^0) = \frac{1}{32\pi} \frac{1}{\sqrt{s} |\mathbf{p}|} \frac{(-g) \pi A(\bar{B}^0 \rightarrow K^- D^+)}{\Gamma(\alpha(0)) \sin \pi \alpha(0)} \int_{-2|\mathbf{p}|^2}^0 e^{\alpha(t) \ln(s/s_0)} dt \quad (25)$$

where we have put [7],  $\alpha(t) \approx \alpha(0)$  in  $\Gamma(\alpha(t))$  and  $\sin \pi \alpha(t)$ . Hence we get, taking  $\alpha(0) \simeq 1/2$  and using linear Regge trajectory:

$$\text{Disc } A(\bar{B}^0 \rightarrow \bar{K}^0 D^0) \approx -\frac{g}{16\sqrt{\pi}} \frac{(s/s_0)^{\alpha_0-1}}{\ln(s/s_0)} A(\bar{B}^0 \rightarrow K^- D^+) \quad (26)$$

We now use dispersion relation to obtain  $A(\bar{B}^0 \rightarrow K^- D^+)$  [6,7]:

$$\begin{aligned} A(\bar{B}^0 \rightarrow \bar{K}^0 D^0)_{\text{FSI}} &= -\frac{g}{16\sqrt{\pi} \ln(m_B^2/s_0)} A(\bar{B}^0 \rightarrow K^- D^+) \times \frac{1}{\pi} \int_{(m_D+m_K)^2}^{\infty} \frac{(s/s_0)^{\alpha_0-1}}{s-m_B^2} \\ &= -\frac{g}{16\sqrt{\pi}} \frac{\sqrt{s_0}}{m_B} \frac{1}{\ln(m_B^2/s_0)} \frac{1}{\pi} \left[ i\pi + \ln \frac{1+x}{1-x} \right] A(\bar{B}^0 \rightarrow K^- D^+) \\ &\equiv \epsilon e^{i\theta} A(\bar{B}^0 \rightarrow K^- D^+) \end{aligned} \quad (27)$$

where in evaluating the dispersion integral we have replaced [7]  $\ln(s/s_0)$  by  $\ln(m_B^2/s_0)$ . In Eq. (27)  $x, \epsilon$  and  $\theta$  are given by

$$\begin{aligned}
x &= \frac{m_D + m_K}{m_B} \simeq 0.447 \\
\epsilon &= -\frac{g}{16\sqrt{\pi}} \frac{s_0}{m_B \ln(m_B^2/s_0)} \sqrt{1 + \frac{1}{\pi^2} \left( \ln \frac{1+x}{1-x} \right)^2} \\
&= -g (2.01 \times 10^{-3}) = 0.20 \\
\theta &= \tan^{-1} \left[ \frac{\pi}{\ln \frac{1+x}{1-x}} \right] \approx 73^\circ
\end{aligned} \tag{28}$$

where we have used  $-g \approx 100$  as given in Eqs. (22) and (23)

Similarly, we obtain

$$A(\bar{B}^0 \rightarrow K^- D^+)_{\text{FSI}} = \epsilon e^{i\theta} A(\bar{B}^0 \rightarrow \bar{K}^0 D^0) \tag{29}$$

For the decays  $\bar{B} \rightarrow \bar{K} \bar{D}$ , we note that except for the decays  $\bar{B}^0 \rightarrow \pi^+ D_s^-$ ,  $B^- \rightarrow \pi^0 D_s^-$ ,  $B^- \rightarrow \eta D_s^-$ , all other decays are either color suppressed or are given by annihilation amplitude. Thus in evaluating rescattering correction, we retain only the dominant amplitudes. From Eqs. (14) and (15), following the same procedure as above, we get

$$A(\bar{B}^0 \rightarrow \bar{K}^0 \bar{D}^0)_{\text{FSI}} = \epsilon e^{i\theta} A(\bar{B}^0 \rightarrow \pi^+ D_s^-) \tag{30}$$

$$\begin{aligned}
A(B^- \rightarrow \bar{K}^0 D^-)_{\text{FSI}, \pi} &= \frac{1}{2} \epsilon e^{i\theta} A(\bar{B}^0 \rightarrow \pi^+ D_s^-) \\
&= -A(B^- \rightarrow K^- \bar{D}^0)_{\text{FSI}, \pi}
\end{aligned} \tag{31}$$

$$\begin{aligned}
A(B^- \rightarrow \bar{K}^0 D^-)_{\text{FSI}, \eta} &= \frac{1}{\sqrt{6}} \epsilon e^{i\theta} \left[ 1 - 2 \frac{\ln(m_B^2/s_0)}{\sqrt{\pi^2 + (\ln(m_B^2/s_0))^2}} e^{i\pi/2} e^{-i\chi} \right] A(\bar{B}^0 \rightarrow \eta D_s^-) \\
&= A(B^- \rightarrow K^- \bar{D}^0)_{\text{FSI}}
\end{aligned} \tag{32}$$

where

$$\chi = \tan^{-1} \frac{\pi}{\ln(m_B^2/s_0)} \approx \tan^{-1}(0.94) \approx \frac{\pi}{4} \tag{33}$$

In order to simplify the calculation, we replace

$$\frac{\ln(m_B^2/s_0)}{\sqrt{(\ln(m_B^2/s_0))^2 + \pi^2}} = 0.727 \approx \frac{1}{\sqrt{2}} \tag{34}$$

Hence, from Eq. (32), using Eqs. (33) and (34), we get

$$\begin{aligned}
A(B^- \rightarrow \bar{K}^0 D^-)_{\text{FSI}, \eta} &= -\frac{i}{\sqrt{6}} \epsilon e^{i\theta} A(\bar{B}^0 \rightarrow \eta D_s^-) \\
&= A(B^- \rightarrow K^- \bar{D}^0)_{\text{FSI}, \eta}
\end{aligned} \tag{35}$$

Now using Eq. (6), and neglecting the contribution of  $A(B^- \rightarrow \bar{K}^0 D^-)$  compared to  $A(\bar{B}^0 \rightarrow \pi^+ D_s^-)$ , we obtain

$$\begin{aligned}
A(B^- \rightarrow \bar{K}^0 D^-)_{\text{FSI}, \pi^0 - \eta} &= \frac{1}{2} \epsilon e^{i\theta} \left( 1 - \frac{i}{3} \right) A(\bar{B}^0 \rightarrow \pi^+ D_s^-) \\
&= \frac{\sqrt{10}}{6} \epsilon e^{i(\theta - \phi)} A(\bar{B}^0 \rightarrow \pi^+ D_s^-)
\end{aligned} \tag{36}$$

$$\begin{aligned}
A(B^- \rightarrow K^- \bar{D}^0)_{\text{FSI}, \pi^0 - \eta} &= -\frac{1}{2} \epsilon e^{i\theta} \left( 1 + \frac{i}{3} \right) A(\bar{B}^0 \rightarrow \pi^+ D_s^-) \\
&= -\frac{\sqrt{10}}{6} \epsilon e^{i(\theta + \phi)} A(\bar{B}^0 \rightarrow \pi^+ D_s^-)
\end{aligned} \tag{37}$$

where

$$\begin{aligned}
\phi &= \tan^{-1} \left( \frac{1}{3} \right) \approx 18^\circ \\
\theta + \phi &= 91^\circ \approx 90^\circ \\
\theta - \phi &= 55^\circ
\end{aligned} \tag{38}$$

Taking into rescattering, Eqs. (2) and (3) are modified

$$A(B^- \rightarrow K^- D^0) = 2f_1 (1 + \epsilon e^{i\theta}) e^{i\delta_1} \tag{39a}$$

$$A(\bar{B}^0 \rightarrow K^- D^+) = f_1 (1 + \epsilon e^{i\theta}) e^{i\delta_1} + f_0 (1 - \epsilon e^{i\theta}) e^{i\delta_0} \tag{39b}$$

$$A(\bar{B}^0 \rightarrow \bar{K}^0 D^0) = f_1 (1 + \epsilon e^{i\theta}) e^{i\delta_1} - f_0 (1 - \epsilon e^{i\theta}) e^{i\delta_0} \tag{39c}$$

$$A(\bar{B}^0 \rightarrow \bar{K}^0 \bar{D}^0) = e^{i\gamma} \left[ 2\bar{f}_1 + \epsilon \bar{f} e^{i(\theta+\bar{\delta})} \right] \tag{40a}$$

$$A(B^- \rightarrow K^- \bar{D}^0) = e^{i\gamma} \left[ \left( \bar{f}_1 + \bar{f}_0 e^{i\bar{\delta}_0} \right) - \frac{\sqrt{10}}{6} \epsilon e^{i(\theta+\phi+\bar{\delta})} \bar{f} \right] \tag{40b}$$

$$A(B^- \rightarrow \bar{K}^0 D^-) = e^{i\gamma} \left[ \left( -\bar{f}_1 + \bar{f}_0 e^{i\bar{\delta}_0} \right) + \frac{\sqrt{10}}{6} \epsilon e^{i(\theta-\phi+\bar{\delta})} \bar{f} \right] \tag{40c}$$

The observables which are significantly affected by rescattering can be easily obtained from Eqs. (39) and (40). From these equations, we get

$$\begin{aligned}
R &= \frac{\Gamma(\bar{B}^0 \rightarrow \bar{K}^0 \bar{D}^0)}{\Gamma(\bar{B}^0 \rightarrow \bar{K}^0 D^0)} \\
&\simeq \frac{4\bar{f}_1^2 \left[ 1 + \epsilon \frac{\bar{f}}{\bar{f}_1} + \epsilon^2 \frac{\bar{f}^2}{4\bar{f}_1^2} \right]}{f_1^2 + f_0^2 - 2f_1 f_0 \cos(\delta_1 - \delta_0)}
\end{aligned} \tag{41}$$

(where in the denominator, we have neglected the terms containing  $\epsilon$ ).

$$\begin{aligned}
R_{1,2} &\equiv \frac{\Gamma(B^- \rightarrow K^- D_{1,2}) + \Gamma(B^+ \rightarrow K^+ D_{1,2})}{\Gamma(B^- \rightarrow K^- D^0)} \\
&= 1 + \frac{1}{4} [r_1^2 + r_0^2 + 2r_1 r_0 \cos \bar{\delta}_0] \mp \cos \gamma [r_1 \cos \delta_1 + r_0 \cos(\delta_1 - \bar{\delta}_0)] - \frac{\sqrt{10}}{6} \epsilon r \cos(\theta + \phi + \bar{\delta} - \delta_1)
\end{aligned} \tag{42}$$

$$\begin{aligned}
\mathcal{A}_{1,2} &\equiv \frac{\Gamma(B^- \rightarrow K^- D_{1,2}) - \Gamma(B^+ \rightarrow K^+ D_{1,2})}{\Gamma(B^- \rightarrow K^- D^0)} \\
&= \mp \sin \gamma \left[ r_1 \sin \delta_1 + r_0 \sin(\delta_1 - \bar{\delta}_0) + \frac{\sqrt{10}}{6} \epsilon r \sin(\theta + \phi + \bar{\delta} - \delta_1) \right]
\end{aligned} \tag{43}$$

where  $D_{1,2} = (D^0 \mp \bar{D}^0)/\sqrt{2}$  are CP-eigenstates with CP = +1, -1 respectively.  $r_1, r_0$  and  $r$  are given by

$$r_1 = \frac{\bar{f}_1}{f_1}, \quad r_0 = \frac{\bar{f}_0}{f_1} \quad \text{and} \quad r = \frac{\bar{f}}{f_1} \tag{44}$$

So far our analysis is general. To proceed further, we note that [9] that these decays are determined by the tree amplitudes  $T(\bar{T})$ , the color suppressed amplitudes  $C(\bar{C})$  and annihilation amplitude  $\bar{A}$ . In terms of these amplitudes

$$f_1 = \frac{G_F}{\sqrt{2}} |V_{cb} V_{us}^*| \frac{1}{2} (T + C) \tag{45}$$

$$f_0 = \frac{G_F}{\sqrt{2}} |V_{cb} V_{us}^*| \frac{1}{2} (T - C) \tag{46}$$

$$\bar{f}_1 = \frac{G_F}{\sqrt{2}} |V_{ub}V_{cs}^*| \frac{1}{2} \bar{C} \quad (47)$$

$$\bar{f}_0 = \frac{G_F}{\sqrt{2}} |V_{ub}V_{cs}^*| \frac{1}{2} (\bar{C} + 2\bar{A}) \quad (48)$$

$$\bar{f} = \frac{G_F}{\sqrt{2}} |V_{ub}V_{cs}^*| \bar{T} \quad (49)$$

Note that in the Wolfenstein representation of CKM matrix [10]:

$$\frac{|V_{ub}V_{cs}^*|}{|V_{cb}V_{us}^*|} = \sqrt{\rho^2 + \eta^2} \quad (50)$$

In the factorization ansatz, these amplitudes are given by [9]

$$\begin{aligned} T &= a_1 f_K F_0^{B-D} (m_K^2) (m_B^2 - m_D^2) \\ C &= a_2 f_D F_0^{B-K} (m_D^2) (m_B^2 - m_K^2) = \bar{C} \\ \bar{T} &= a_1 f_{D_s} F_0^{B-\pi} (m_{D_s}^2) (m_B^2 - m_\pi^2) \\ \bar{A} &= a_1 f_B F_0^{D-K} (m_B^2) (m_D^2 - m_K^2) \end{aligned} \quad (51)$$

where  $F_0(t)$  is scalar form factor for  $B$  to  $P$  transition ( $P = D, K$  or  $\pi$ ). For these form factors, we use the following values

$$F_0^{B-K} (m_D^2) \approx F_0^{B-\pi} (m_{D_s}^2) \simeq 0.22 \quad (52)$$

and [9]

$$F_0^{B-D} (m_K^2) \approx 0.587 \quad (53)$$

Using [9],  $f_D \simeq 200$  MeV,  $f_{D_s} \simeq 240$  MeV,  $f_B \simeq 180$  MeV,  $f_K = 158$  MeV and  $a_2/a_1 \simeq 0.26$ , we get

$$\begin{aligned} r_1 &= \frac{\bar{f}_1}{f_1} = \sqrt{\rho^2 + \eta^2} \frac{\bar{C}}{C + T} \approx 0.04 \\ r_0 &= \frac{\bar{f}_0}{f_1} = \sqrt{\rho^2 + \eta^2} \frac{\bar{C} + 2\bar{A}}{C + T} \approx 0.08 \\ r &= \frac{\bar{f}}{f_1} = \sqrt{\rho^2 + \eta^2} \frac{\bar{T}}{C + T} \approx 0.41 \end{aligned} \quad (54)$$

where we have

$$\sqrt{\rho^2 + \eta^2} = 0.36 \quad (55)$$

We now discuss the consequences of our main results given in Eqs. (41) and (43). First we note that it follows from Eq. (14), that  $\bar{\delta}$  can be put equal to zero (no elastic scattering for  $\pi D_s^-$ ). Hence, from Eq. (41), we get

$$R = R_0 \left[ 1 + \epsilon \bar{r} \cos \theta + \frac{\epsilon^2 \bar{r}^2}{4} \right] \quad (56)$$

where

$$R_0 = \frac{4\bar{f}_1^2}{f_1^2 + f_0^2 - 2f_1 f_0 \cos(\delta_1 - \delta_0)} \quad (57)$$

is the branching ratio in the absence of rescattering. From Eqs. (47), (49) and (50), we get

$$\bar{r} \equiv \frac{\bar{f}}{f_1} = \frac{2\bar{T}}{C} \simeq 5.00 \quad (58)$$

Hence we obtain using  $\epsilon = 0.20$  and  $\theta = 73^\circ$ ,

$$R = 1.54R_0 \quad (59)$$

We now discuss the consequence of Eq. (43) i.e. direct CP-violation in  $\bar{B}$  decays. Let us first assume that final state interactions are taken care of by the phases induced by rescattering. Hence we put  $\delta_1$  and  $\delta_1 - \bar{\delta}_0$  equal to zero. Then, we get from Eq. (43)

$$\begin{aligned} \mathcal{A}_{1,2} &= \mp \sin \gamma \left[ \frac{\sqrt{10}}{6} \epsilon r \sin(\theta + \phi) \right] \\ &= \mp \sin \gamma \left[ \frac{\sqrt{10}}{6} \epsilon r \right] \\ &= \mp \sin \gamma (0.043) \end{aligned} \quad (60)$$

since  $\theta + \phi \approx 90^\circ$ .

In general however

$$\mathcal{A}_{1,2} = \mp \sin \gamma \left[ 0.04 \sin \delta_1 + 0.08 \sin(\delta_1 - \bar{\delta}_0) + 0.04 \sin\left(\frac{\pi}{2} - \delta_1\right) \right] \quad (61)$$

But the structure of Eqs. (8), (9), (16) and (17) implies that  $\bar{\delta}_0$  has the same sign as  $\delta_0$ , it is therefore reasonable to conclude that  $\delta_1 - \bar{\delta}_0$  has the same sign as  $\delta_1 - \delta_0$ . Since  $\delta_0 = -\delta_1$ , it follows that (since  $\delta_1$  has positive sign), that  $\mathcal{A}_{1,2}$  should be atleast  $\mp (0.043) \sin \gamma$ . The phases  $\delta_1, \bar{\delta}_0$  are expected to be small. As an example, let us take  $\delta_1 = 13^\circ, \delta_0 = -13^\circ = \bar{\delta}_0$ , then we get

$$\mathcal{A}_{1,2} = \mp \sin \gamma (0.086) \quad (62)$$

The direct CP-violation is an important consequence of the standard model. But in the absence of final state interactions, this parameter is zero. Our analysis shows that even if strong phases  $\delta$ 's are negligible, the rescattering gives a finite value for  $\mathcal{A}_{1,2}$  which may be experimentally measureable in future experiments. Even if our estimate of  $\epsilon$  is off by a factor 2,  $\mathcal{A}_{1,2}$  will still have at least the value  $\mp 0.023 \sin \gamma$ . Thus unless in an unlikely case that  $\bar{\delta}_0$  has the same sign as  $\delta_1$  and much greater than  $\delta_1$ , One may conclude that direct asymmetry parameter may lie in the range

$$\mp 0.023 \sin \gamma \leq \mathcal{A}_{1,2} \leq \mp 0.086 \sin \gamma \quad (63)$$

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